Title: Detective Slope – An Investigation of the Slopes of Lines and Shapes

Brief Overview:

This learning unit is designed for students to investigate the definition of slope and slopes of parallel and perpendicular lines. Students will be introduced to special parallelograms by applying the concept of slope using Geometer's Sketchpad.

NCTM Content Standard:

Geometry

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Algebra

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.

Grade/Level:

Grades 9-12, Geometry and Algebra 1.

Duration/Length:

Three class periods, 50-60 minutes in length.

Student Outcomes:

Students will:

- Define slope.
- Use the features of Geometer's Sketchpad to plot points, construct lines, measure slope, and measure angles.
- Identify lines with positive and negative slopes.
- Classify the slopes of horizontal and vertical lines.
- Identify the slopes of parallel and perpendicular lines.
- Discover properties of special parallelograms by applying the concept of slope.

Materials and Resources:

- Geometer's Sketchpad or TI-92 plus/ Voyage200 calculator
- Graph paper
- Tape measure
- Student worksheets (activity sheets, homework sheets and assessment sheets)
- Teacher Notes
- Textbook

How to Use These Lessons:

Lesson 1 is designed for the teacher with access to a computer or calculator with Geometer's Sketchpad that can be seen by the class. Lesson 2 and Lesson 3 are designed for the teacher and students with access to a computer or calculator with Geometer's Sketchpad. If this is not possible, the lessons can be adapted for the teacher to demonstrate the activities while students participate verbally.

Development/Procedures:

Lesson 1

Preassessment/Launch –Students will be divided into groups of four or fewer. Each group will measure two steps of a staircase in the hallway and record their measurements and answer the questions on ***Worksheet 1****: Investigating Stairs*. (The height of the steps is the rise. The length of each step is the run.) After the student investigation, the teacher discusses the idea of steepness. This discussion should lead into the definition of slope as the steepness of a line. Slope is also defined as rise divided by run. Continue the idea of slope as rise divided by run with ***Worksheet 2***.

Teacher Facilitation/ Student Application –

- Using Geometer's Sketchpad (see ***Worksheet 2:*** *Teacher Resource Sheet –Creating Geometer's Sketchpad Rise Run demonstration activity* for setup), demonstrate the possible values (negative, positive, zero and undefined) for the slope of the line by moving points ***A*** and ***B*** around. Discuss with the students their observations. Let the students state verbally conjectures about finding slope. (Possible conjectures: if the line rises, or goes uphill, the line has positive slope; horizontal lines have zero slope; vertical lines have no slope (or undefined); if the line falls (downhill) the slope is negative.)
- Then have students work on the ***Worksheet 3:*** *Investigating Slope*. Once they have finished, ask students what formula they came up with and discuss their answers.

- Be sure the students' investigation led them to the slope formula. Given two points A(x_1, y_1) and B(x_2, y_2), the slope m of $\overline{AB}$ can be written as $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Finally, give students the homework ***Worksheet 4:*** *Investigating Slopes Revisited.*

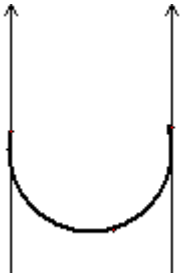
Embedded Assessment – To check for understanding give students the following problem.
Plot point A (0, 1) and point B (-1, 5) on graph paper and sketch the line connecting them.
Find the slope of line AB using rise and run:
rise =
run =
slope =

Find the slope of line AB using the slope formula:
x_1 =
y_1 =
x_2 =
y_2 =

Reteaching/Extension – To re-teach or help students remember zero versus undefined slope use the picture below

Horizontal lines have

Vertical lines have:



Undefined Slope

Investigating Stairs

Each group will measure two steps of a staircase. Measure the height and depth of each step with a ruler. Record your measurements. Sketch the staircase (two steps) and label each step with the appropriate measurement.

Questions:

1. How could the measurements be changed to make the staircase steeper?

2. How could the measurements be changed to make the staircase less steep?

3. What is the vertical distance between the first and last steps?

4. What is the horizontal distance between the first and last steps?

5. What is the ratio of your answers the # 3 and #4? How does this ratio compare to the ratio of the height and depth of a single step?

Investigating Stairs
Teacher's Guide

Each group will measure two steps of a staircase. Measure the height and depth of each step with a ruler. Record your measurements. Sketch the staircase (two steps) and label each step with the appropriate measurement.

Questions:

1. How could the measurements be changed to make the staircase steeper?

   ANSWERS WILL VARY

2. How could the measurements be changed to make the staircase less steep?

   ANSWERS WILL VARY

3. What is the vertical distance between the first and last steps?

   ANSWERS WILL VARY

4. What is the horizontal distance between the first and last steps?

   ANSWERS WILL VARY

5. What is the ratio of your answers the # 3 and #4?

   ANSWERS WILL VARY

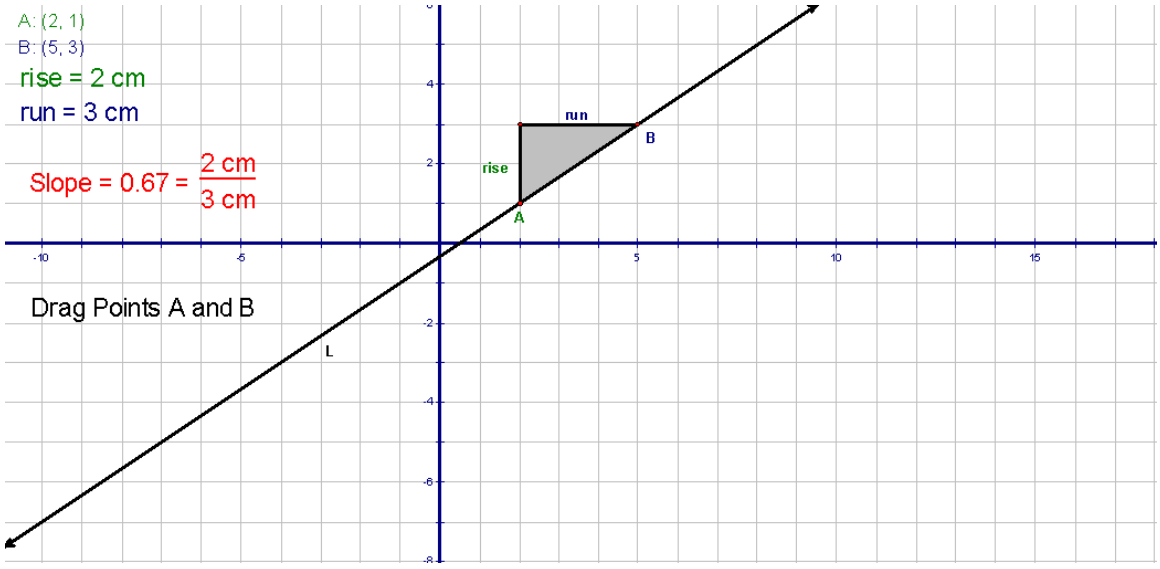
6. How does this ratio from # 5 compare to the ratio of the height and depth of a single step?

   THEY ARE EQUAL

Teacher Resource Sheet - Creating Geometer's Sketchpad Rise Run demonstration activity

If you have already installed the extra Geometer's Sketchpad files use the ***Rise Run*** activity to introduce slope to the class. To do this, go to the ***Exploring Algebra*** file, double-click on the ***2_Lines*** file and double-click on ***Rise Run***.

If these extra files are not available, take 5 to 10 minutes to follow these steps to create the Rise Run activity shown below. The directions are written for Geometer's Sketchpad on a personal computer but may be modified for use on a TI 92-Plus or TI Voyager 200.



1. Create a New Sketch in Sketchpad
2. Click on Edit -> Preferences…
   Change the Units of *Distance* to **cm** and the Precision to **Units**
   Change the Precision of *Other (Slope, Ratio,…)* to **Units**
3. Click on Graph -> Show Grid
4. Click on Graph -> Snap Points
5. Highlight the points (0,0) and (1,0) and click on Display -> Hide Points
6. Using the button (and NOT Graph ->Plot Points), plot a point on (2,1). Highlight point, select textbox, and label point ***A***.
7. Using the button (and NOT Graph ->Plot Points), plot a point on (5,3). Highlight point, select textbox, and label point ***B***.
8. Construct line ***L*** through points ***A*** and ***B***
9. Highlight point ***A*** and the x-axis and click on Construct -> Perpendicular Line
10. Highlight point ***B*** and the y-axis and click on Construct -> Perpendicular Line
11. Construct a point on the intersection of the perpendicular lines
12. Construct a line segment between the intersection of the perpendicular lines and point ***A***

13. Construct a line segment between the intersection of the perpendicular lines and point ***B***
14. Highlight the two perpendicular lines and click on Display -> Hide Perpendicular Lines
15. Label the last horizontal segment. Highlight line, select textbox, and label it ***run***.
16. Label the last vertical segment. Highlight line, select textbox, and label it ***rise***.
17. Highlight the three vertices of the triangle and click on Construct -> Triangle Interior
18. Highlight points ***A*** and ***B*** and click on Measure -> Coordinates
19. Highlight the ***rise*** and ***run*** segments and click on Measure -> length
20. Highlight line ***L*** and click on Measure -> slope
21. Highlight the slope measurement and click on Edit -> Properties…
    Under the *Value* tab change the *Precision* to **hundredths**
22. Create a new text box and type in the following using the keyboard:

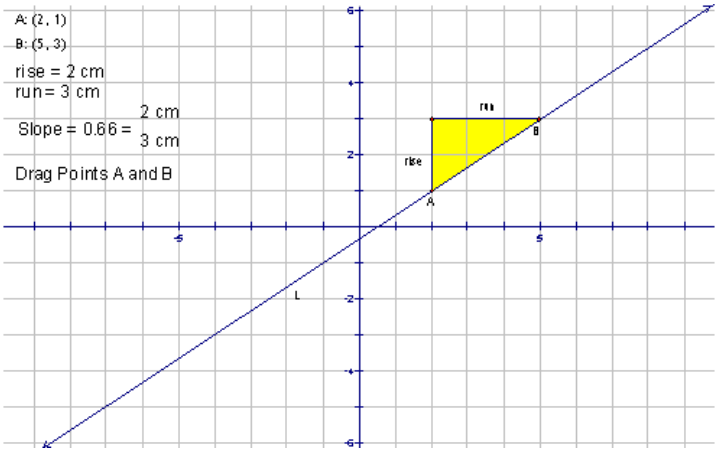
$$= \text{Slope} = \{1\} = \frac{\{2\}}{\{3\}}$$

    To construct the fraction click the  button and then the  button. Then using the keyboard, type {2} in the numerator and {3} in the denominator
23. With the new text box highlighted, highlight the ***slope***, ***rise***, and ***run*** measurements, IN THAT ORDER!
    Click on Edit -> Merge Text
    $\text{Slope} = 0.67 = \frac{2\text{cm}}{3\text{cm}}$ should appear on the screen.
24. Highlight the ***Slope L*** measurement and the

$$= \text{Slope} = \{1\} = \frac{\{2\}}{\{3\}}$$

    text box and click on Display ->Hide Text Objects
25. Create a new text box and type: **Drag Points A and B**

CONGRATULATIONS!!!! You should have the diagram below. Move points ***A*** and ***B*** to demonstrate slope concepts. Use this slope demonstration in your classes for years to come!!



Optional: Improve the appearance of the slope demonstration by adding color and formatting text using the directions below.

1. Change the colors of all labels and measurements associated with the ***rise*** GREEN
    i. Label of line ***rise***
    ii. Label of point ***A***
    iii. Coordinates of point ***A*** => A: (2,1)
    iv. ***rise*** measurement => rise = 2 cm
2. Change the colors of all labels and measurements associated with the ***run*** BLUE
    i. Label of line ***run***
    ii. Label of point ***B***
    iii. Coordinates of point ***B*** => B: (5,3)
    iv. ***run*** measurement => run = 3 cm
3. Change the color of the text $\text{Slope} = 0.67 = \frac{2\,\text{cm}}{3\,\text{cm}}$
4. Change the color of the triangle interior to gray.
5. Increase the size of these four text boxes:
    i. rise = 2 cm
    ii. run = 3 cm
    iii. Drag Points A and B
    iv. $\text{Slope} = 0.67 = \frac{2\,\text{cm}}{3\,\text{cm}}$

Investigating Slopes

1) Plot the given points and draw the line through the points. Draw the slope triangle to indicate the rise and run. Then find the slope.
   a) A(-1, 3) and B(2, 4)
   b) C(0, 6) and D(6, 0)
   c) E(-4, -1) and F(6, -1)
   d) G(2, 5) and H(2, 7)

2) Complete the table using the information above.

Points	Rise	Run	Slope
A(-1, 3) and B(2, 4)	1	3	$\frac{1}{3}$
C(0, 6) and D(6, 0)			
E(-4, -1) and F(6, -1)			
G(2, 5) and H(2, 7)			

3) Use parts 1 and 2 to answer the following questions.

   a) Given the points (-1, 3) and (2, 4), what is the rise of the line passing through those points? Now find the difference of the y-coordinates. Compare the numerical values for the rise and the difference of the y-coordinates.

   b) What is the run of the line passing through the two points? Now find the difference of the x-coordinates. Compare the numerical values for the run and the difference of the x-coordinates.

   c) COMPLETE: rise = difference of the _____ coordinates.
   run = difference of the _____ coordinates.

   d) The difference of the y-coordinates using algebraic symbols can be written as $y_2 - y_1$. Write the difference of the x-coordinates using algebraic symbols.

   e) Use your previous answers and the definition of slope to complete the following statement.

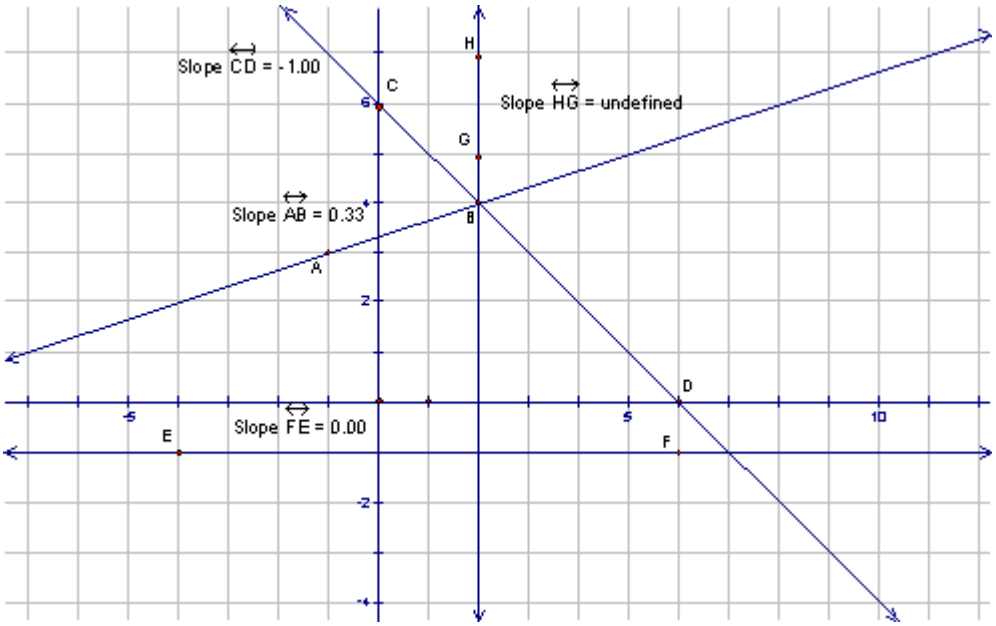
$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{difference of}}{\text{difference of}} = \_\_\_\_\_$$

   f) In general the formula for the slope of a line is m = _____.

Investigating Slopes
Teacher's Guide

1) Plot the given points and draw the line through the points. Draw the slope triangle to indicate the rise and run. Then find the slope. Leave your answers as fractions.
   a) A(-1, 3) and B(2, 4)
   b) C(0, 6) and D(6, 0)
   c) E(-4, -1) and F(6, -1)
   d) G(2, 5) and H(2, 7)

Possible student answers



2) Complete the table using the information above.

Possible student answers

Points	Rise	Run	Slope
A(-1, 3) and B(2, 4)	1	3	$\frac{1}{3}$
C(0, 6) and D(6, 0)	**6**	**-6**	$\frac{\mathbf{6}}{\mathbf{-6}}$ **= -1**
E(-4, -1) and F(6, -1)	**0**	**-10**	$\frac{\mathbf{0}}{\mathbf{-10}}$ **= 0**
G(2, 5) and H(2, 7)	**-2**	**0**	$\frac{\mathbf{-2}}{\mathbf{0}}$ **= undefined**

3) Use parts 1 and 2 to answer the following questions.
   a) Given the points (-1, 3) and (2, 4), what is the rise of the line passing through those points? **1** Now find the difference of the y-coordinates. **(4 -3 = 1)** Compare the numerical values for the rise and the difference of the y-coordinates. **The values are the same.**

   b) What is the run of the line passing through the two points? **(3)** Now find the difference of the x-coordinates. **(2 – (-1) = 3)** Compare the numerical values for the run and the difference of the x-coordinates. **The values are the same.**

   c) COMPLETE: rise = difference of the **y** coordinates.
      run = difference of the **x** coordinates.

   d) The difference of the y-coordinates using algebraic symbols can be written as $y_2 - y_1$. Write the difference of the x-coordinates using algebraic symbols. **$(x_2 - x_1)$**

   e) Use the definition of slope and your previous work to complete the following statement.

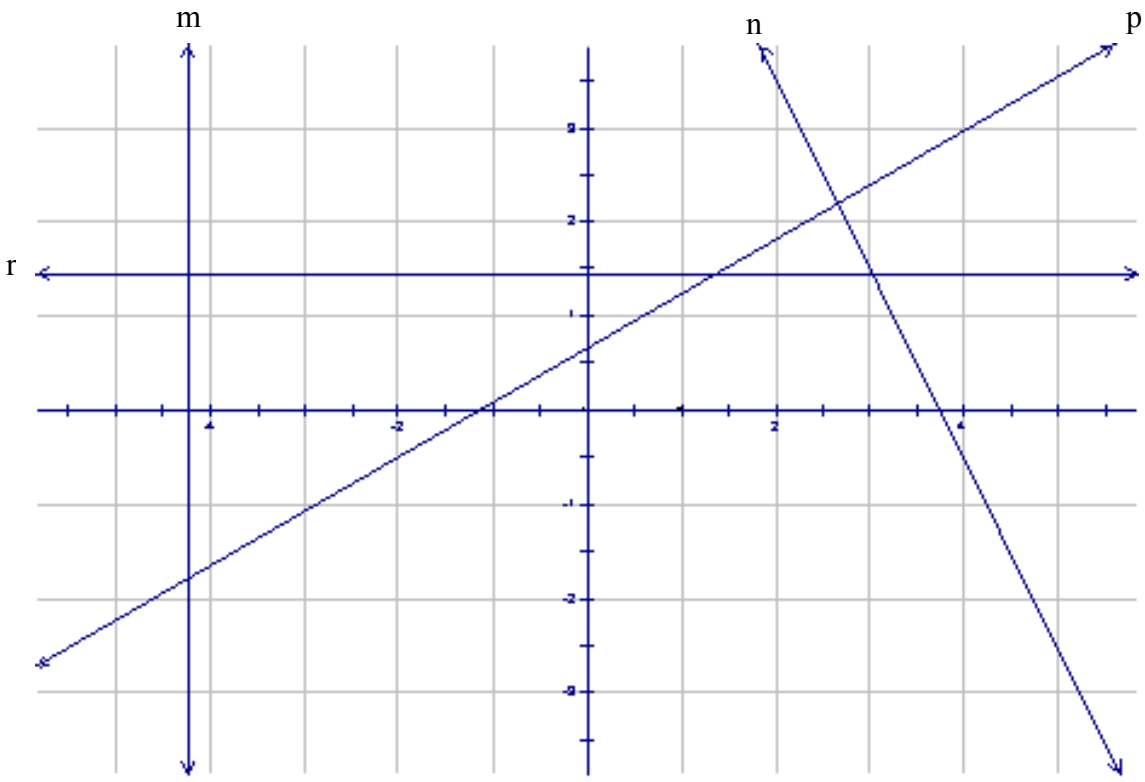
$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{difference of } \mathbf{y}-\mathbf{coordinate\ s}}{\text{difference of } \mathbf{x}-\mathbf{coordinate\ s}} = \frac{y_2 - y_1}{x_2 - x_1}$$

   f) In general the formula for the slope of a line is $\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1}$.

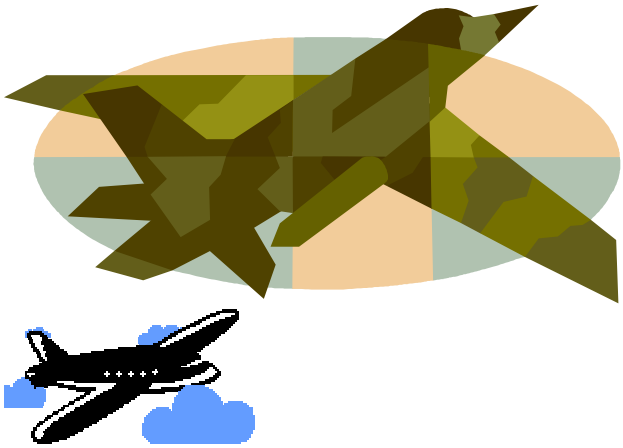
Investigating Slope Revisited

1) Sketch each line.
   a) The line passes through the points (-2, 1) and (1, 2).
   b) The line passes through the point (0, -5) and (4, -6).

2) Name each line in the figure whose slope is positive, negative, zero or undefined.



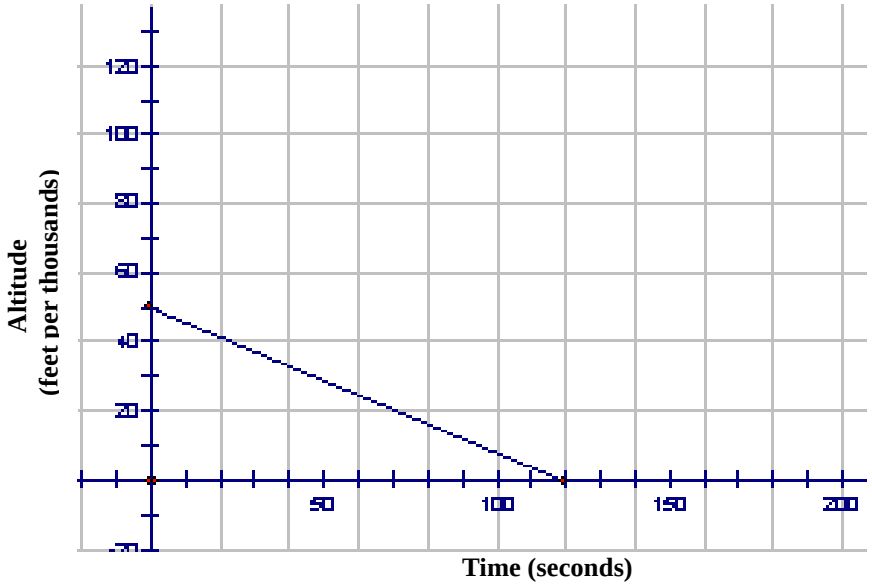
3)



Look at a Real-World Problem

The graph below shows the altitude of an airplane from the time the wheels are lowered (time = 0 seconds) to when the plane lands. Find the slope of the line. Explain what the slope means in this situation.

Find any two points on the graph. Use the points to find the slope.

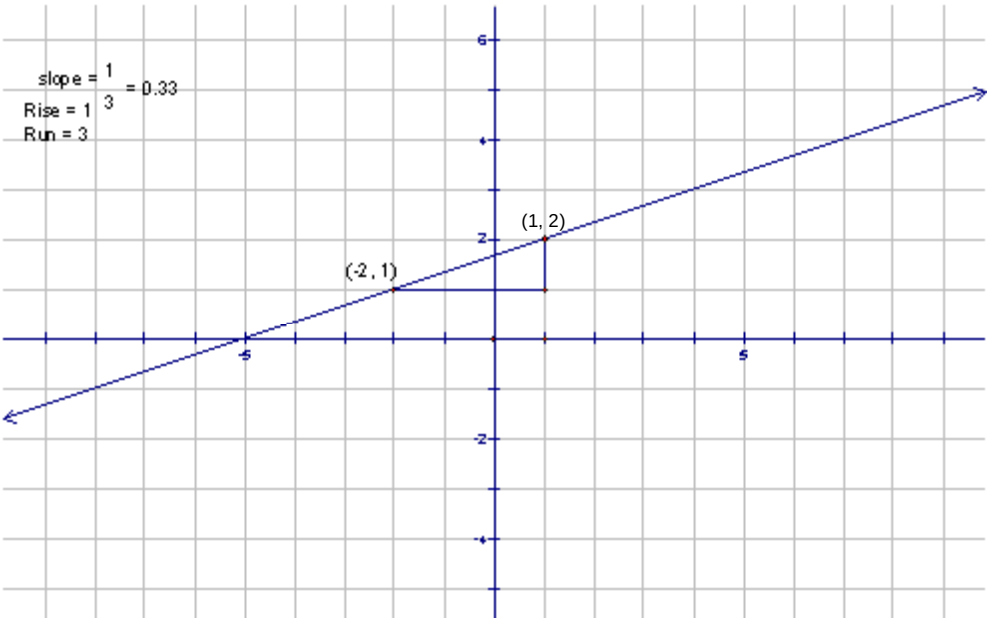


Critical Thinking --- Suppose a similar graph is made for another airplane and the slope of the line is 14. What would the slope tell you in this situation?

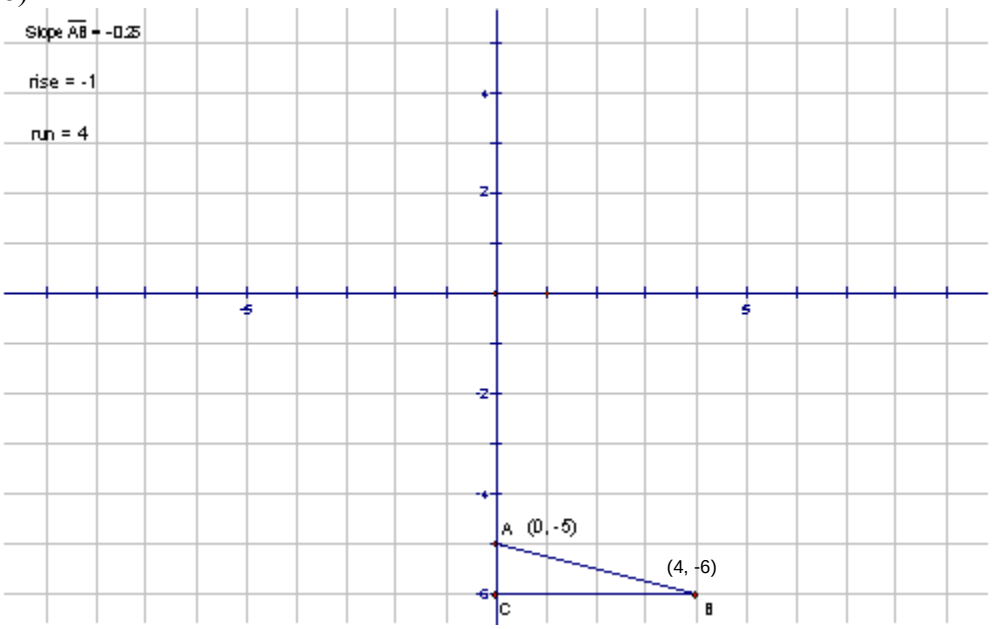
Investigating Slope Revisited
Teacher's Guide

1) Sketch each line.
   a) The line passes through the points (-2, 1) and (1, 2).
   b) The line passes through the point (0, -5) and (4, -6).

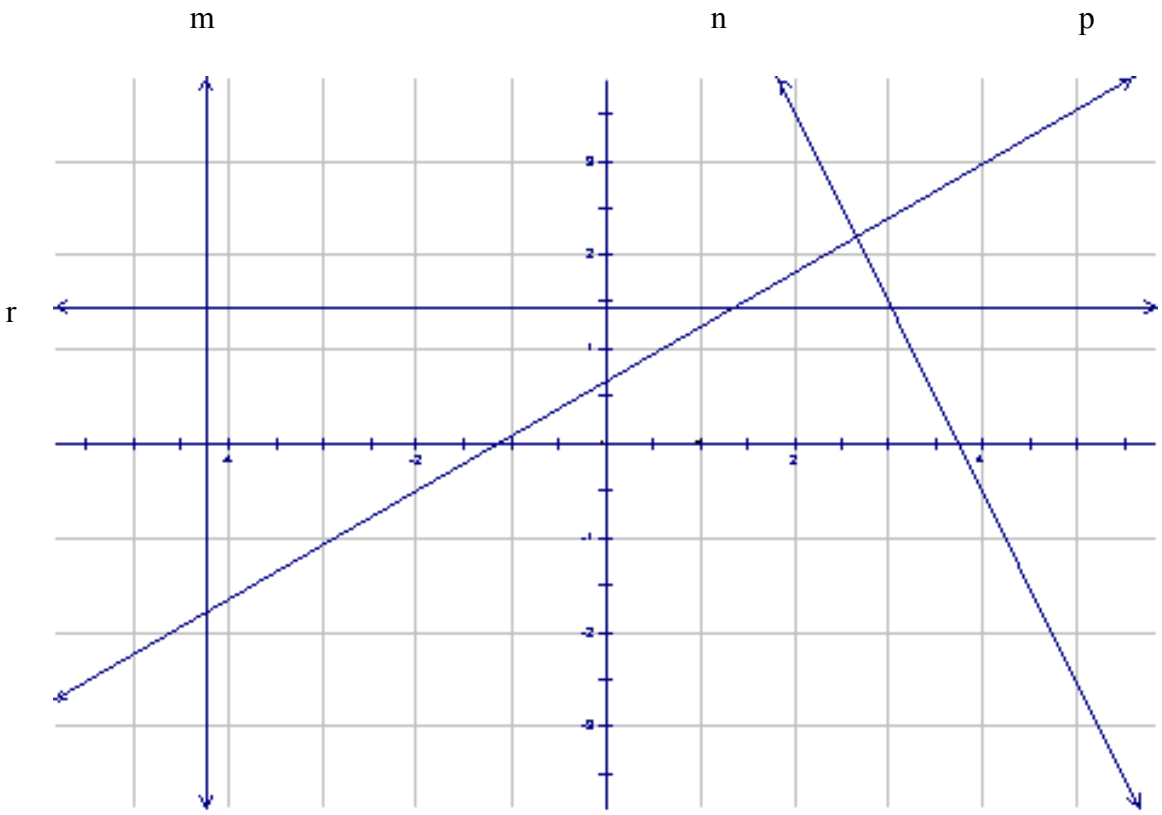
a)



b)

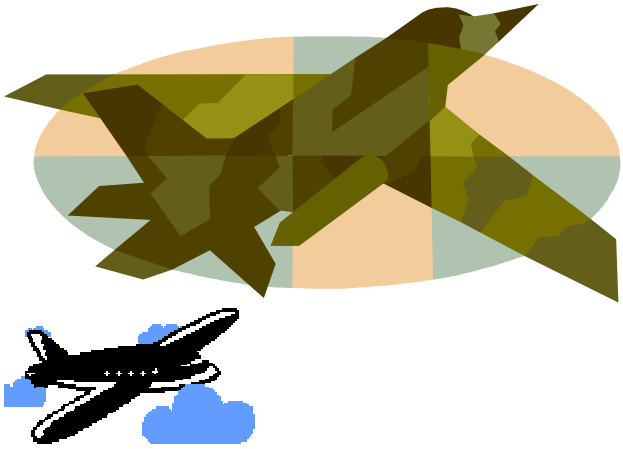


2) Name each line in the figure whose slope is positive, negative, zero or undefined.



Slope of line p is positive.
Slope of line n is negative.
Slope of line r is 0.
Slope of line m is undefined.

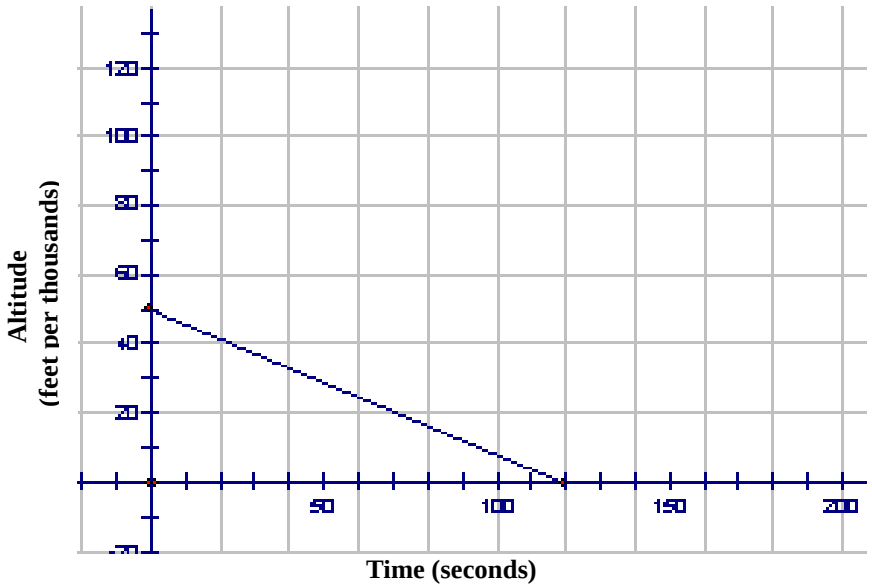
3)



Look at a Real-World Problem

The graph below shows the altitude of an airplane from the time the wheels are lowered (time = 0 seconds) to when the plane lands. Find the slope of the line. Explain what the slope means in this situation.

Find any two points on the graph. Use the points to find the slope.



$$\text{Slope} = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}}$$
$$= \frac{-50{,}000}{120}$$
$$= -416.66$$

The slope of the line is -416.66. The airplane is descending 416.66 feet per second.

Critical Thinking --- Suppose a similar graph is made for another airplane and the slope of the line is 14. What would the slope tell you in this situation?

The airplane ascends at 14 feet per second. (14 ft/s).

Lesson 2 **Preassessment/ Launch -**

a) Have two groups of four or five students stand on both sides of a rectangular table to demonstrate parallel lines.
b) Have two groups of four or five students make a form of the letter 'T' with their bodies to demonstrate perpendicular lines.
c) Teach the concept that perpendicular lines form right angles.

Teacher Facilitation – Review the ideas of parallel and perpendicular lines. Introduce the geometric symbols ∥ and ⊥. Discuss ***Worksheet 1***: *Investigating Slopes of Perpendicular and Parallel Lines* directions especially the directions for using the Geometer's Sketchpad to complete the worksheet. After students have completed the worksheet, go over the conjectures students came up with and discuss the slopes of parallel and perpendicular lines. For homework students will complete ***Worksheet 2***: *Investigating Parallel and Perpendicular Lines Revisited*.

Student Application – Students will use Geometer's Sketchpad to investigate the slopes of parallel and perpendicular lines using ***Worksheet 1***. They will write conjectures about the slopes of parallel lines and perpendicular lines. Students will be able to apply their conjectures to the homework problems accompanying this lesson.

Embedded Assessment – Students are asked to graph other lines that are parallel or perpendicular to the given line in order to verify their conjectures. Students should notice that all parallel lines have the same slope and the product of the slopes of all perpendicular is negative one.

Reteaching/Extension – For those who have not completely understood the lesson, they are encouraged to redo the steps of the activity with a peer; choosing a different set of points.

For those who have understood the lesson, have them verify the conjectures formed in the worksheet by plotting points on graph paper and finding the slope using the slope formula.

Investigating Slopes of Perpendicular and Parallel Lines

Directions: Follow the steps below in Geometer's Sketchpad (computer software).

1) Start with NEW SKETCH (function key F1).
2) Click on (function key F6) GRAPH -> SHOW GRID. Click GRAPH ->PLOT POINTS. Enter (-3, 2) then click PLOT. Enter (3, -1), click PLOT and then click DONE.
3) Label (-3, 2) point A and (3, -1) point B.
4) Highlight points A and B and click on (function key F3) CONSTRUCT -> LINE.
5) Click GRAPH ->PLOT POINTS. Enter (2, 1) and then click PLOT. Then click DONE. Label (2, 1) point C.
6) Highlight line AB and point C and click on CONSTRUCT -> PARALLEL LINE.
7) Construct point D at (-2, 3).
8) Highlight line AB and line CD and click on (function key F5) MEASURE -> SLOPE

Part 1: Use steps 1 through 8.

a. What are the slopes of line AB and line CD? (write in fraction form)

b. How do the two slopes compare?

c. Construct at least two other lines (your choice) that are parallel to line AB.

d. What conjecture can you make about the slopes of parallel lines? Be sure your conjecture matches with all your observations.

Part 2: Use steps 1 through 5.

Start with a NEW SKETCH. Repeat steps 1 through 5 listed above. Then highlight line AB and point C and click on CONSTRUCT -> PERPENDICULAR LINE.

a. Calculate the slopes of the two lines. Write your answer as a fraction.

b. How do the two slopes compare?

c. Construct at least two other lines (your choice) that are perpendicular to line AB.

d. What conjecture can you make about the slopes of perpendicular lines? Check your results with your conjecture about perpendicular lines.

points	slope	slope of parallel line	slope of perpendicular line
A(-3, 2) and B(3, -1)			

e. Using a different set of points, verify your conjectures about slopes of parallel and perpendicular lines.

Investigating Slopes of Perpendicular and Parallel Lines

Teacher's Guide

Directions: Complete this activity using the Geometer's Sketchpad or Voyage 200/TI 92 +. **(These problems can be done using graph paper and pencil.)**

1) Start with NEW SKETCH.
2) Click on GRAPH -> SHOW GRID. Click GRAPH ->PLOT POINTS. Enter (-3, 2) then click PLOT. Enter (3, -1), click PLOT and then click DONE.
3) Label (-3, 2) point A and (3, -1) point B.
4) Highlight points A and B and click on CONSTRUCT -> LINE.
5) Click GRAPH ->PLOT POINTS. Enter (2, 1) and then click PLOT. Then click DONE. Label (2, 1) point C.
6) Highlight line AB and point C and click on CONSTRUCT -> PARALLEL LINE.
7) Construct point D at (-2, 3).
8) Highlight line AB and line CD and click on MEASURE -> SLOPE

Part 1: Use steps 1 through 8.

a. What are the slopes of line AB and line CD? (write in fraction form) **(-½,-½)**

b. How do the two slopes compare? **(the same)**

c. Construct at least two other lines (your choice) that are parallel to line AB.

d. What conjecture can you make about the slopes of parallel lines? Be sure to check the slopes of the lines you constructed with your conjecture.
The slopes of parallel lines are equal.

Part 2: Use steps 1 through 5.
Start with a NEW SKETCH. Repeat steps 1 through 5 listed above. Then highlight line AB and point C and click on CONSTRUCT -> PERPENDICULAR LINE.

a. Calculate the slopes of the two lines. Write your answer as a fraction. **(-½, 2)**

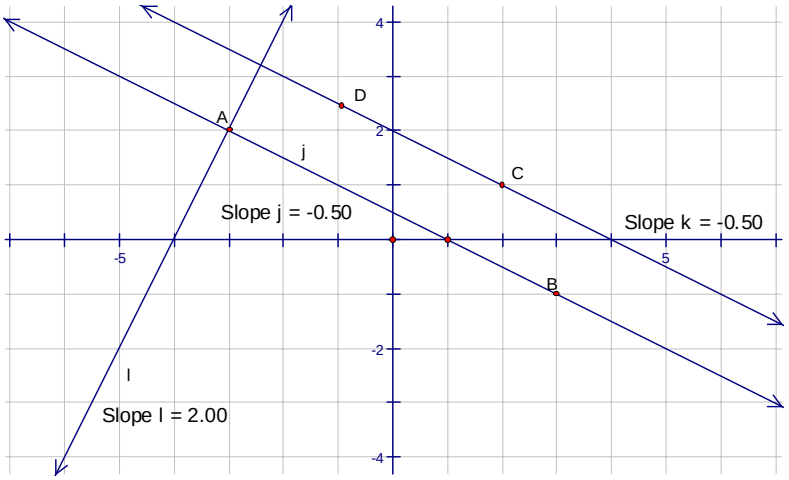
b. How do the two slopes compare? **Negative reciprocals**

c. Construct at least two other lines (your choice) that are perpendicular to line AB.

d. What conjecture can you make about the slopes of perpendicular lines? Check your results with your conjecture about perpendicular lines.
The product of the slopes of perpendicular lines is equal to –1.

points	slope	slope of parallel line	slope of perpendicular line
A(-3, 2) and B(3, -1)	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{2}{1}$

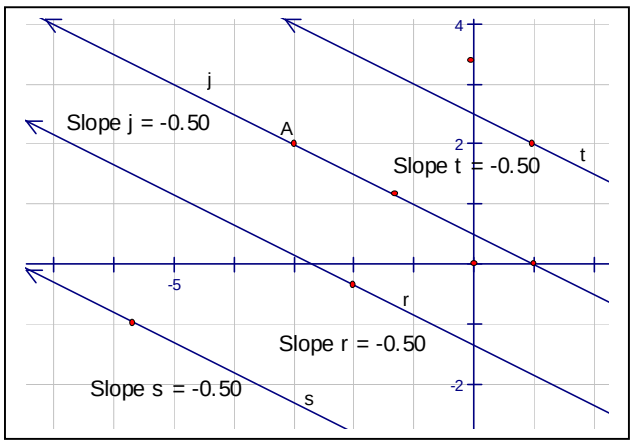
Student answers.



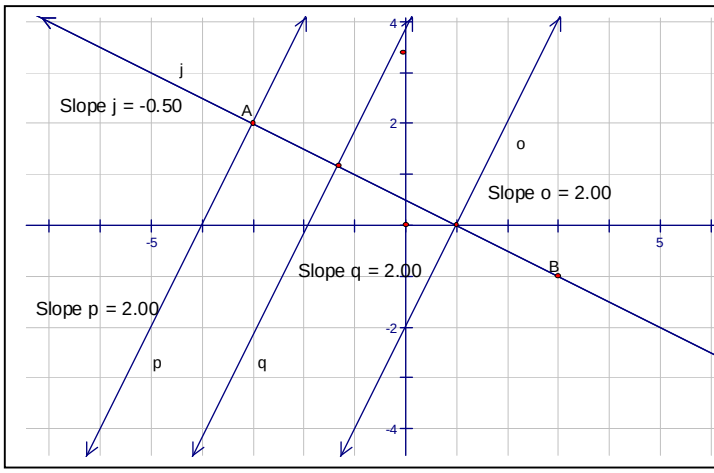
e. Using a different set of points, verify your conjectures about slopes of parallel and perpendicular lines.

Note to teacher if students have problems continuing on their own, give them points to plot.

Possible student answers for parallel lines.



Possible student answers for Perpendicular lines.



Investigating Parallel and Perpendicular Lines Revisited

1) The slopes of two lines are given below. Determine whether the lines are parallel, perpendicular or neither.

Slopes	$\parallel$, $\perp$ or neither
$\frac{1}{2}$, -2	
-5, -5	
7, $\frac{1}{7}$	
$\frac{3}{4}$, $-\frac{4}{3}$	

2) a) Plot the points and join the points from A to D in order.
A(-1, -3), B(0, -4), C(2, -2), D(1, -1)

b) What is the slope of $\overline{AD}$? of $\overline{BC}$?

c) What is the slope of $\overline{AB}$? of $\overline{DC}$?

d) What is the slope of $\overline{AB}$? of $\overline{BC}$?

e) What is the slope of $\overline{AD}$? of $\overline{DC}$?

f) What can you say about $\overline{AD}$ and $\overline{DC}$?

g) What can you say about $\overline{AB}$ and $\overline{DC}$?

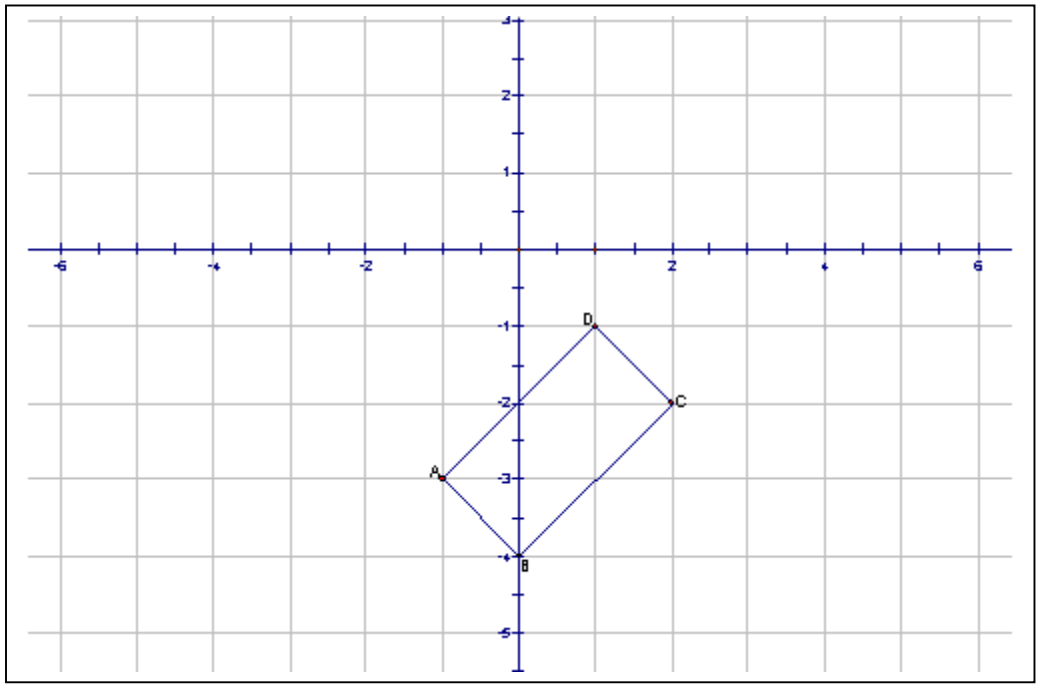
h) What is this figure called?

Investigating Parallel and Perpendicular Lines Revisited
Teacher's Guide

1) The slopes of two lines are given below. Determine whether the lines are parallel, perpendicular or neither.

Slopes	$\|\|$, $\perp$ or neither
$\frac{1}{2}$, -2	**perpendicular**
-5, -5	**parallel**
7, $\frac{1}{7}$	**neither**
$\frac{3}{4}$, $-\frac{4}{3}$	**perpendicular**

2) a) Plot the points and join the points from A to D in order.
A(-1, -3), B(0, -4), C(2, -2), D(1, -1)



b) What is the slope of $\overline{AD}$? of $\overline{BC}$?

Slope of $\overline{AD}$ = 1

Slope of $\overline{BC}$ = 1

c) What is the slope of $\overline{AB}$? of $\overline{DC}$?

Slope of $\overline{AB}$ = -1

Slope of $\overline{DC}$ = -1

d) What is the slope of $\overline{AB}$? of $\overline{BC}$?

Slope of $\overline{AB}$ = -1

Slope of $\overline{BC}$ = 1

e) What is the slope of $\overline{AD}$? of $\overline{DC}$?

Slope of $\overline{AD}$ = 1

Slope of $\overline{DC}$ = -1

f) What can you say about $\overline{AD}$ and $\overline{DC}$?

$\overline{AD}$ and $\overline{DC}$ are perpendicular .

g) What can you say about $\overline{AB}$ and $\overline{DC}$?

$\overline{AB}$ and $\overline{DC}$ are parallel.

h) What is this figure called?

Rectangle

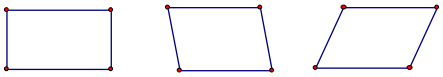
Lesson 3

Preassessment -Ask students review questions over finding slope, definition of a polygon, definition of parallel and perpendicular lines, and the basics for using Geometer's Sketchpad (if students will each have access to the software)

Launch – Instruct student's to create a rectangle using Geometer's Sketchpad. If students' are not comfortable enough with Sketchpad to do this on their own give the students the ***Worksheet 1:*** *Constructing a Rectangle with Sketchpad*.

Teacher Facilitation – When students have finished constructing their rectangles, present the definition of a quadrilateral and a parallelogram.
A *Quadrilateral* is a polygon with four sides
A *Parallelogram* is a quadrilateral with both pairs of opposite sides parallel
Have students draw on paper three parallelograms that look different.

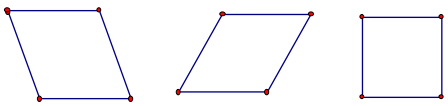
Sample:



Check students' work. Now students will begin the ***Worksheet 2:*** *Investigating Rectangles* worksheet. When they are finished, go over the answers with them. Ask a couple of students to give their definition of a rectangle and write the best one on the board. Next, teach students the definition of a rhombus.

A *Rhombus* is a parallelogram with four congruent sides
Have students draw on paper three rhombuses that look different.

Sample:



Check students' work. It's important the sides look congruent (as close as a student can draw by hand), to show understanding. Finally, students will discover that a square is a parallelogram that is both a rectangle and a rhombus

with the embedded assessment.

Student Application – Students will complete ***Worksheet 2*** using the rectangle created with Geometer's Sketchpad.

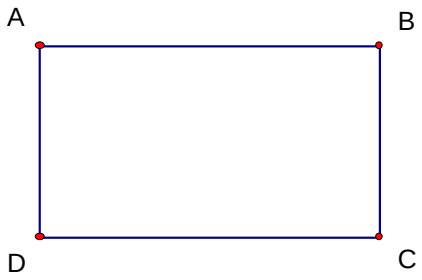
Embedded Assessment – Have students' construct a parallelogram that is both a rectangle and a rhombus using Geometer's Sketchpad. Instruct students' to verify that the quadrilateral is a parallelogram, rectangle and a rhombus by measuring the slopes and lengths of each side and the included angles. Ask students' to name the shape they have constructed (a square). Then give students the definition of a square.
A *Square* is a parallelogram with four congruent sides and four right angles

Reteaching/Extension – For those who have not completely understood the lesson or to take students to the next step give students a copy of the ***Worksheet 3:*** *Designs with Parallelograms.*

Constructing a Rectangle with Sketchpad

1. Open a New Sketch in Geometer's Sketchpad
2. Construct a horizontal segment ***AB***
3. Highlight segment ***AB*** and point ***B*** and click on Construct->Perpendicular Line
4. Highlight segment ***AB*** and point ***A*** and click on Construct->Perpendicular Line
5. Construct point ***C*** on the perpendicular line that goes through point ***B***.
6. Highlight point ***C*** and segment ***AB*** and click on Construct -> Parallel Line
7. Construct point ***D*** on the intersection of the perpendicular lines through point ***A*** and Point ***C***
8. Highlight point ***B***, point ***C***, point ***D***, and then point ***A*** and click on Construct -> Segments
9. Highlight the three lines ***BC***, ***CD***, and ***AD*** and click on Display->Hide Lines

You should see a rectangle similar to the one below.



You can Click and drag point B to rotate and change the size of Rectangle ABCD.

Investigating Rectangles

Directions: Draw Rectangle ABCD using Geometer's Sketchpad and use it to complete the following exercises.

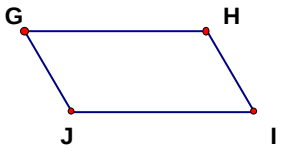
1. How do you know the quadrilateral you have constructed is a rectangle?

2. Is Rectangle ABCD a Parallelogram? Justify by using Sketchpad.

3. Click and drag the vertices of Rectangle ABCD to rotate it and change its size. Is Rectangle ABCD still a parallelogram, why or why not?

4. Can you construct a rectangle that is not a parallelogram? Explain your answer.

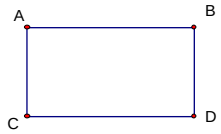
5. Is Parallelogram GHIJ below a rectangle? Explain.



6. Based on your answers to # 4 and # 5 write a possible definition for a rectangle.

Investigating Rectangles
Teacher's Guide

Directions: Draw Rectangle ABCD using Geometer's Sketchpad and use it to complete the following exercises. **Sample Rectangle:**



1. How do you know the quadrilateral you have constructed is a rectangle?

   Sample Answer: It is a quadrilateral with four right angles.

2. Is Rectangle ABCD a Parallelogram? Justify by using Sketchpad.

   Sample Answer: Yes, slopes of opposite sides are equal.

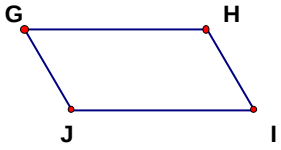
   Slope $\overline{AB}$ = 0.00 Slope $\overline{AC}$ = undefined
   Slope $\overline{CD}$ = 0.00 Slope $\overline{BD}$ = undefined

3. Click and drag the vertices of Rectangle ABCD to rotate it and change its size. Is Rectangle ABCD still a parallelogram? Why or why not?

   Yes, it is still a parallelogram because the slopes are still equal so the lines are still parallel.

4. Can you construct a rectangle that is not a parallelogram? Explain your answer.
   No, if it's a rectangle then both pairs of opposite sides are parallel. All rectangles are parallelograms.

5. Is Parallelogram GHIJ below a rectangle? Explain.



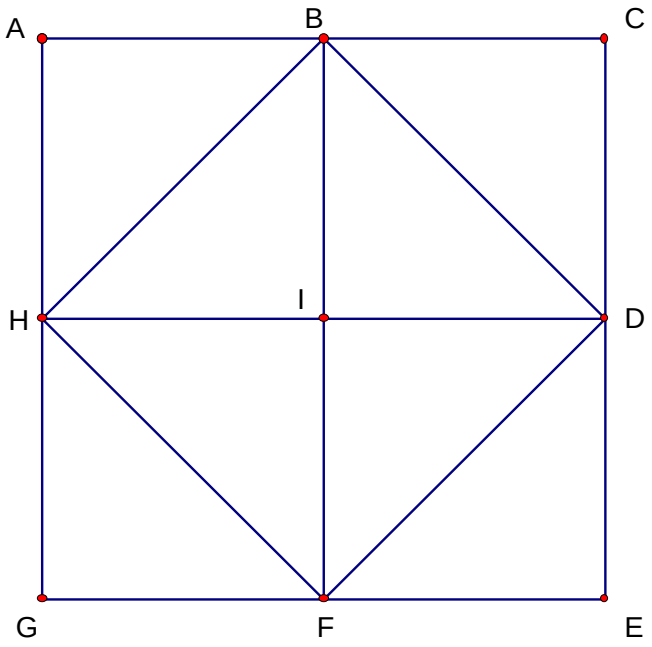
   No, it does not have four right angles.

6. Based on your answers to # 4 and # 5 write a possible definition for a rectangle.

   A rectangle is a parallelogram with four right angles.

Designs with Parallelograms

Directions – Measure the segments and angles in the design below with a protractor to complete the exercises.



1. How many squares are in the figure above?

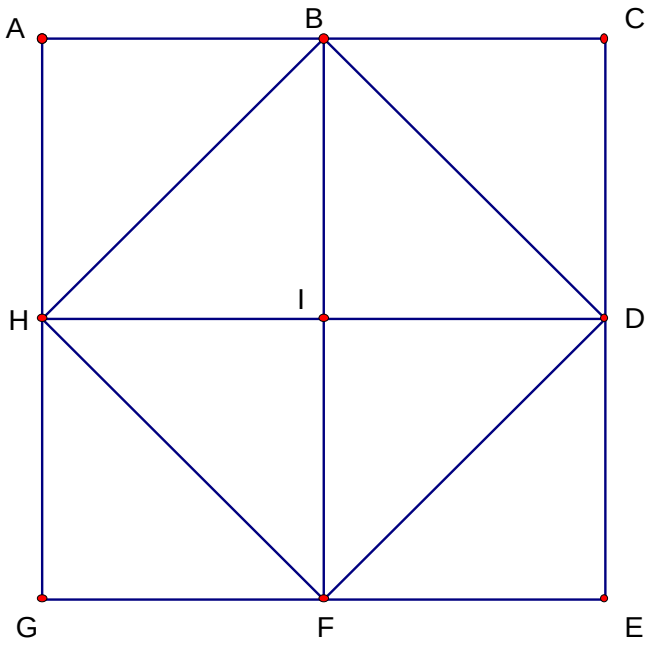
2. How many rectangles are in the figure above?

3. How many rhombuses are in the figure above?

4. How many parallelograms are in the figure above?

Designs with Parallelograms
Teacher's Guide

Directions - Measure the segments and angles in the design below with a protractor to complete the exercises.



1. How many squares are in the figure above?

   6

2. How many rectangles are in the figure above?

   10

3. How many rhombuses are in the figure above?

   6

4. How many parallelograms are in the figure above?

   10

Summative Assessment:

To assess the progress students, use the *Solving for Slopes of Lines and Properties of Parallelograms* quiz.

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References:

Algebra: Tools for a Changing World 1998. Prentice Hall, New Jersey.

Solving for Slopes of Lines and Properties of Parallelograms

5 points each

1. A geometry term for the incline of a ramp is _____.

2. On the coordinate plane the slope can be measured as _____ over _____.

3. Slope can be represented by the letter _____.

4. The formula for the slope of a line is _____.

5. If the slopes of two lines are equal, then the lines are _____.

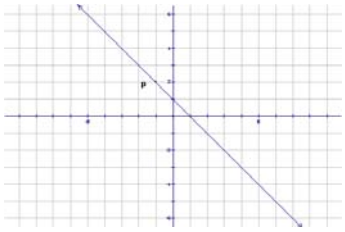
6. If two lines are perpendicular, then the product of their slopes is _____.

7. A polygon with four sides is a _____.

8. The adjacent sides of a rectangle or a square are _____ lines.

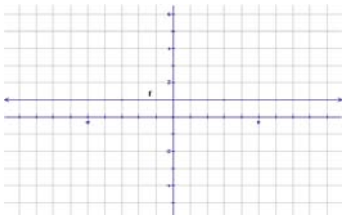
9. Opposite sides of a rectangle, parallelogram, or a square are _____ lines.

10. Which correctly describes the slope of the line p below?



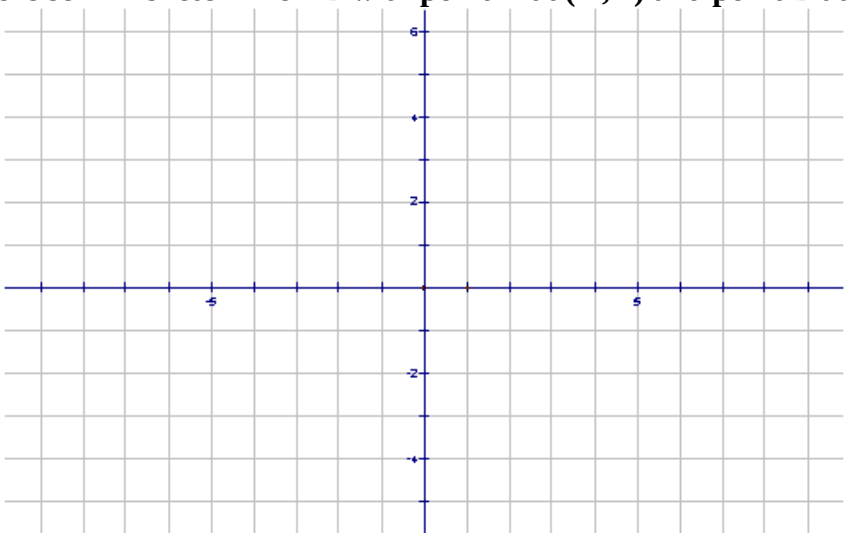
a. positive
b. negative
c. zero
d. undefined

11. Which correctly describes the slope of the line r below?



a. positive
b. negative
c. zero
d. undefined

For exercise # 12 sketch line AB with point A at (-1, 4) and point B at (2, 3)



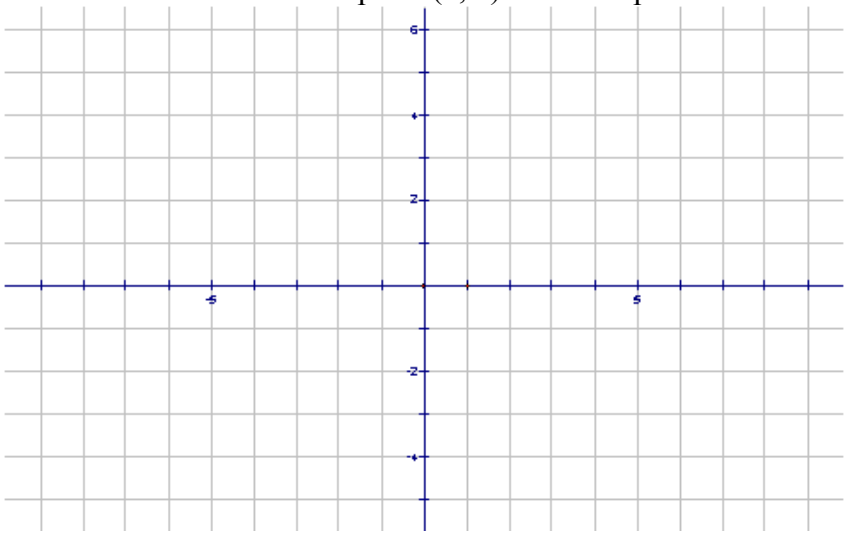
12. Using your sketch above, what are the rise, run and slope of line AB?

rise =

run =

slope =

13. Sketch the line that crosses the point (2, 5) with a slope of -1.



14. Fill in the blanks below for the points (-1, -1) and (4, 2).

x_1 =
y_1 =
x_2 =
y_2 =
slope (m) =

15. Fill in the blanks below for the points (-4, -1) and (6, -1).

$x_1 =$
$y_1 =$
$x_2 =$
$y_2 =$
slope (m) =

16. Circle **two** slopes below that would form parallel lines.

I. $m = 2$ II. $m = \frac{1}{2}$ III. $m = 2$ IV. $m = -\frac{1}{2}$

17. Circle **two** slopes below that would form perpendicular lines.

I. $m = 2$ II. $m = \frac{1}{2}$ III. $m = 2$ IV. $m = -\frac{1}{2}$

18. Sketch a parallelogram that is not a rectangle.

19. What is one difference between a rhombus and a rectangle?

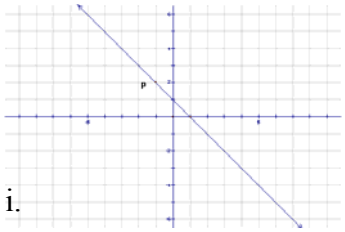
20. What is another name for a quadrilateral that is a rectangle, rhombus, and a parallelogram?

Solving for Slopes of Lines and Properties of Parallelograms

Teacher's Guide

5 points each

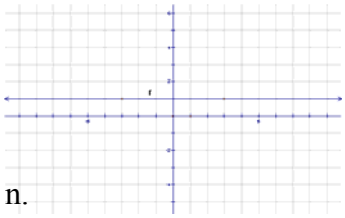
1. A geometry term for the incline of a ramp is **slope**.
2. On the coordinate plane the slope can be measured as **rise** over **run**.
3. Slope can be represented by the letter **m**
4. The formula for the slope of a line is $m = \frac{y_2 - y_1}{x_2 - x_1}$
5. If the slopes of two lines are equal, then the lines are **parallel**
6. If two lines are perpendicular, then the product of their slopes is **-1**
7. A polygon with four sides is a **quadrilateral**
8. The adjacent sides of a rectangle or a square are **perpendicular** lines.
9. Opposite sides of a rectangle, parallelogram, or a square are **parallel** lines.
10. Which correctly describes the slope of the line p below?



i.

e. positive
f. negative
g. zero
h. undefined

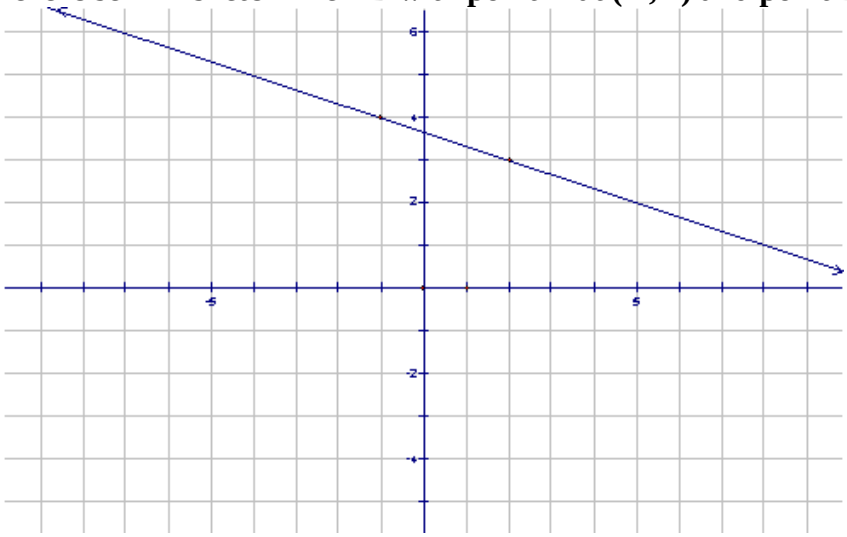
11. Which correctly describes the slope of the line r below?



n.

j. positive
k. negative
l. zero
m. undefined

For exercise # 12 sketch line AB with point A at (-1, 4) and point B at (2, 3)



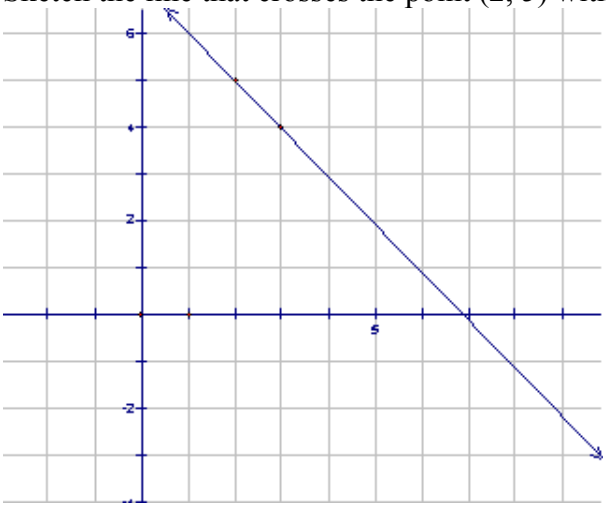
12. Using the sketch above, what are the rise, run and slope of line AB?

rise = ***1***

run = ***-3***

slope = $-\frac{\mathbf{1}}{\mathbf{3}}$

13. Sketch the line that crosses the point (2, 5) with a slope of -1.



14. Fill in the blanks below for the points (-1, -1) and (4, 2).

x_1 = ***-1***

y_1 = ***-1***

x_2 = ***4***

y_2 = ***2***

slope (m) = $\frac{\mathbf{3}}{\mathbf{5}}$

15. Fill in the blanks below for the points (-4, -1) and (6, -1).

x_1 = ***-4***
y_1 = ***-1***
x_2 = ***6***
y_2 = ***-1***
slope (m) = ***0***

16. Circle **two** slopes below that would form parallel lines.

I. m = 2 II. m = $\frac{1}{2}$ ***III. m = 2*** IV. m = $-\frac{1}{2}$

17. Circle **two** slopes below that would form perpendicular lines.

I. m = 2 II. m = $\frac{1}{2}$ III. m = 2 ***IV. m =*** $-\frac{\mathbf{1}}{\mathbf{2}}$

18. Sketch a parallelogram that is not a rectangle.

19. What is one difference between a rhombus and a rectangle?

A rectangle must have four right angles and a rhombus does not have to.
Or
A rhombus must have four congruent sides and a rectangle does not have to.

20. What is another name for a quadrilateral that is a rectangle, rhombus, and a parallelogram?

Square